General Discussion

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General discussion

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Some aspects of creep in concrete

First I wish to thank the organizers for allowing me to offer a few comments in relation to another widely used engineering material which exhibits creep behaviour, namely concrete. This material exhibits a number of features which are similar to those described already for materials such as rocks and metals, while at the same time offering some distinctly dissimilar features. For example, the dominant state of stress in concrete structures is compressive and in consequence it is the compressive creep which is of importance in most practical situations. Concrete exhibits deviatoric and volumetric creep and thus differs from many other materials which show zero creep dilatation. The creep rate is related linearly to stress and continuously decreases with time under load; it increases with increase of temperature. Creep occurs at all stress levels: there is no threshold stress level which must be exceeded for creep to occur.

The creep of concrete has three levels of importance:

(1) The creep behaviour originates at the submicroscopic level in the gel and hydration products of the cement phase of the material, and here water plays a dominant rôle in determining the magnitude of creep observed in experiments.

(2) Creep at the macroscopic, or engineering, level is observed as an integrated effect which represents the behaviour of a heterogeneous material consisting of both active and passive material constituents. The observed behaviour is taken to represent an isotropic property at the engineering level.

(3) In the engineering structure, creep, although isotropic, may be influenced by environmental factors such as humidity and temperature and may thus become a non-homogeneous property throughout the volume of the structure. When this happens gross departures of behaviour from the predictions of normal elastic analyses are encountered. Experiments conducted on concrete beams and portal frame structures have indicated that supporting actions not only change with time due to creep, but will in many cases exhibit a change of sense, whether they be bending moments or forces at the foundations.

Experimental creep data reveal at least two separable components of creep strain. One is of the viscous flow type and the other reflects a delayed elastic strain response to changes of stress. The first is temperature-dependent and may conveniently be normalized with respect to stress and temperature for use in analysis. At elevated temperatures it dominates over the delayed elastic component, which is essentially temperature-independent, and for this reason the latter component may be either ignored as a significant creep component, or simply included with the normal elastic strain response by modifying the elastic modulus. After normalization the strain rate equation may be written

$$\mathrm{d}\epsilon_c/\mathrm{d}t = \sigma f(T)\, g(t), \tag{1}$$

where $f(T)$ and $g(t)$ represent respectively the normalizing creep-temperature function and an ageing function of time, and σ is the applied stress.

At constant stress and temperature equation (1) represents the creep rate and its variation with time. A comparable but simpler equation may be developed when the normalized creep strain itself, $\epsilon_c/\sigma f(T)$, is taken to represent a pseudo-time parameter, t', say. It then follows that the creep rate, $\dot{\epsilon}_c$, with respect to t', has the form

$$\dot{\epsilon}_c = \sigma f(T). \tag{2}$$

In other words, the creep rate is related linearly to stress, to some function of temperature, $f(T)$, and is constant with respect to t'. This last property permits considerable simplification to be achieved in the formulation of analytical techniques in structural analysis for creep of concrete in non-homogeneous situations. Conversion to real time is made at the end of the analysis by reference to the normalized creep–real time curve for the material.

Use of the formulation of equation (2) and its corresponding equivalent representation in three dimensions has allowed calculations to be made in terms of the rate at which work is done on the structure by the external loads undergoing displacements caused by creep. The power $\dot{W}$ associated with the external loads is equated to the rate of change $\dot{U}$ of stored energy in the material and the power $\dot{D}$ dissipated due to creep. Then

$$\dot{W} = \dot{U}+\dot{D}. \tag{3}$$

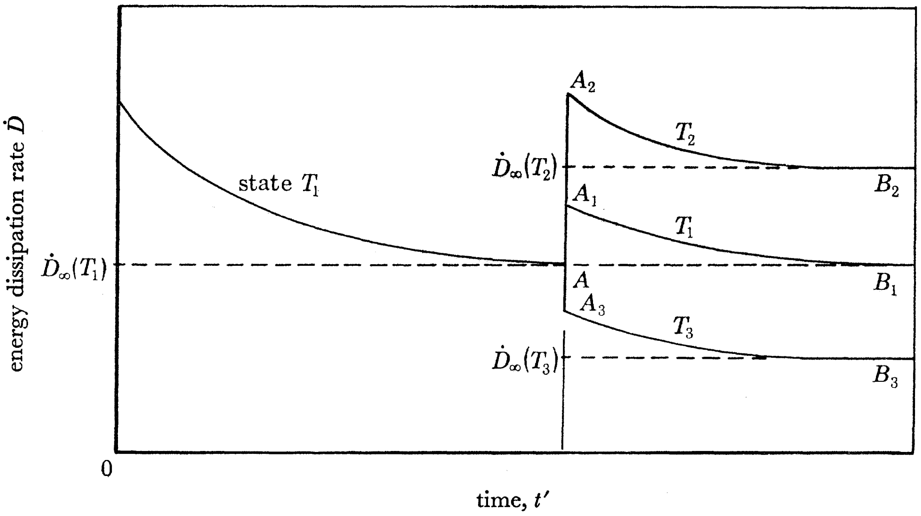

FIGURE 1. Variation with time t', of power dissipated in creep.

The two quantities $\dot{U}$ and $\dot{D}$ represent the integrated sum of corresponding density quantities in the structure; they thus vary in space and time. In addition, there is an interchange between the stored energy and dissipated energy, even when the applied loads are constant in time. Analyses indicate that a preferred long-term solution exists and that this corresponds to a condition in which the power dissipated in creep is a minimum with respect to the pseudo-time parameter. At this time the stored energy becomes a constant in time but does not represent a minimum as in the case of a wholly elastic system.

Further observations indicate that it is possible to obtain by direct calculation the limiting state of stress in a structure undergoing non-homogeneous creep by taking advantage of the knowledge that the energy dissipation rate due to creep alone in the steady-state stress condition (see England 1966) is a minimum with respect to all variations of stress from the true state (see England 1968).

An interesting feature of the rate at which energy is dissipated due to creep is shown in figure 1, where $\dot{D}$ is plotted against t' for concrete subjected to sustained temperatures and loads. At point A, it is assumed that $\dot{D} \approx \dot{D}_\infty$, the steady-state energy dissipation rate and that at this time changes to the system are introduced which alter the state of internal stress. Two cases are considered:

(*a*) A self-equilibrating system of stress is introduced without change of temperature. The resulting response of $\dot{D}$ is as shown by the curve A_1–B_1.

(*b*) A change to the state of non-uniform temperature is introduced. This causes a change to the internal stresses and alters the local creep behaviour of the material in a non-homogeneous manner. The subsequent behaviour with reference to $\dot{D}$ has two possible forms. These are shown by the curves A_2–B_2 and A_3–B_3. In each case the new preferred state for $\dot{D}$ is of smaller magnitude than the value of $\dot{D}$ immediately following the change of temperature, irrespective of whether this preferred state, denoted by points B_2 and B_3, lies above or below the point A for the initial set of conditions.

The behaviour illustrated above forms a basis for the understanding of concrete at the engineering level and allows the laws of mechanics to be used in the formulation of new theorems, which in turn, permit reliable engineering predictions to be made by direct calculation in areas hitherto examined only by historical step-by-step forms of solution.

In conclusion I leave by asking this question: is it likely that an energy dissipation formulation, to the flow and recrystallization problem of creep, can lead to a smooth steady-state creep rate as observed in experiments and to suitable predictive methods of analysis for other materials discussed at this meeting?

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The materials considered at this meeting seem to fall into two categories: those which undergo creep under a tensile stress (mainly metals) and those which creep under a compressive stress (principally rocks). Now, there exists a material which creeps under both these stresses, indeed under all practical states of stress, and which can be tested in a like manner in the laboratory: concrete. Tests on creep of concrete in compression and in tension and measurements of recovery of creep after removal of either of these stresses are helpful in elucidating the mechanism of creep.

An important feature of the creep of concrete is that it occurs at all temperatures, at least in the practical range of something like -50 to $+400$ °C, and under any stress; there is thus no threshold stress or temperature, although creep is of course a function of both these variables.

We do not have a satisfactory understanding of the mechanism of creep of concrete but we know that at temperatures below about 80° C the presence of adsorbed water in the cement paste is necessary. Strictly speaking, it is only cement paste that creeps, the aggregate not doing so at stresses which are practicable in concrete. So, although concrete is a two-phase material, the major part of creep deformation occurs not at boundaries between phases or between grains but within the cement phase.

Creep of concrete under cyclic stresses is of interest not only for structural design purposes but also in the study of the phenomena involved in creep and recovery; the activation energy approach has been very helpful. Work on this has just been published (Neville & Hurst 1977; Hurst & Neville 1977) as a supplement to a book (Neville 1970).

Finally, I should like to thank the organizers of the meeting for the opportunity they gave me to look at other materials. I am sure this is good for all of us.

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Creep studies and planetary problems

Although my contribution to the discussion was made in response to the first paper of the meeting presented by Professor Weertman, having now listened to the rest of the discussion I should like to broaden somewhat the scope of my original comments to cover what I regard as a serious omission. My comments to Professor Weertman were concerned with a particular misconception of the boundary value problem, involving the creep behaviour of *in situ* planetary material, that is posed by Earth and indeed all the planets. It is very important that one's thoughts about planetary creep be guided by some preliminary understanding of the way this boundary value problem is posed, for without it I have noticed that any discussion of 'The creep of the Earth' is likely to be replaced by a description of observations properly subsumed under the heading 'Laboratory studies of creep in rocks or minerals'. In turn these will generate a host of unanswerable, if not irrelevant, questions about the 99.9999...% of planetary material that is not accessible to similar observation. Let me say at once that any criticism is not intended for those who study rock creep for its intrinsic interest, but at the extremely naïve views that some of them entertain about the relevance of their results to planetary scale problems.

As a group, engineers are probably far more aware than most geophysicists that quite unforeseen problems of material science can arise when one is trying with all one's skill simply to scale up a prototype system by a factor of ten in linear dimensions, so I may well be preaching to the converted in saying that I would profoundly distrust the reliability of using results obtained on a system that was, say, 10^8 times smaller than the one of interest *even if I knew that size was the only respect in which the two systems differed.* Of course, there is absolutely no hope of settling even this latter point for a planet, but some very careful experimenter may well ask whether such distrust is justified or even helpful when, unlike some he could mention, he is at least producing 'hard data' about Earth properties. To answer this, it is necessary to examine exactly what has happened when someone claims to have produced hard data about a macroscopic property of matter. Although the experimenter may believe he has observed and measured, for example, a viscosity, what he has done in fact is to use a solution of equations containing the concept of a viscous continuum to connect limited ranges of two quantities that he interprets as a rate of strain and stress. He will not waste much time testing whether the fitted value of viscosity is a truly intensive quantity because he is usually convinced before he starts the experiment that there is a viscosity loitering inside his specimen waiting to be measured. Such imagery is quite harmless until one asks the question: what is the relation between stress and rate of strain for a specimen of the same material *n* times larger (or smaller)? The 'hard data' man with his realist views about material properties will have no doubt that the answer is to be found by assigning the same viscosity to all points of the new system's interior and, of course, provided n is not too big or too small this often works. However, by using the matter-of-fact description of the experimenter's activities, one can see that what is really involved in this question is whether the equations whose solution was used to connect the observables 'stress' and 'rate of strain' in the original experiment can be dynamically scaled. What makes me so pessimistic about using laboratory creep data in planetary scale problems is that their substitution into the appropriate equations of motion leads to equations that have no scaling laws. In other words, the empirical data are already telling us that a different relation will hold between the same pair of observables if the size is changed, and that the concept of property data that are 'hard' in the sense of being applicable to a material, whenever or however it is found, is no better than a dangerous half truth. Perhaps to reassure those geophysicists who think I am proposing to throw alway all laboratory data, I should say that I see this difficulty mainly in connection with the dynamical rather than the static properties of materials. While the latter can probably be safely seen as labels attached to a material whenever it turns up, the dynamical properties must be viewed with the equations to which they belong as forming an abstract model connecting the observables of systems. In this abstraction many will recognize that we are already using a systems approach to dynamical problems on a laboratory scale and, in fact, what I am suggesting is that it is only the realist misapprehension about material properties that makes people think they can do otherwise in a planetological situation. However, for the same reasons that engineers find moderately scaled models useful, if not perfect, I have much higher hopes of finding an integrated view of dynamical processes in different planets than of also explaining laboratory phenomena with exactly the same theory.

Although there are a number of planetary observations of deformation on a large scale in whose interpretation a creep resistance in the form of an effective viscosity has been assigned to the interior (Cathles 1975), the most basic problem involving creep uses the concept of a system undergoing an internal heat transfer process to connect observations of surface heat flow and the large-scale secular deformation of near surface rocks. The heat sources are visualized as only very slowly changing, temperature-independent, radiogenic sources distributed throughout the interior. This problem is more basic in the sense that if, instead of treating effective viscosity as an adjustable parameter (as is done in the other problems), we introduce the assumption that *in situ* planetary material also has a very temperature sensitive creep resistance, we may show that within wide limits of choice for such creep functions, the mean values of an effective viscosity controlling large-scale deformation are quite closely regulated by the heat-transfer process itself. My comment to Professor Weertman was made because he assumes, as do so many others for purely historical reasons, that it is the temperature rather than the effective viscosity that is directly regulated by the planetary heat-transfer process. Those who are further interested in the self regulation of planetary creep resistance can pursue the matter through a recent article of mine (Tozer 1977), but I would make the following comments to illustrate the importance of some preliminary understanding of this boundary value problem before getting too deeply involved in discussion of the creep process. It is the effective viscosity $\sigma/\dot{e}$ for large-scale deformation that is directly regulated by the heat transfer process, and I may add that the regulated value of *ca.* 10^{21} P† agrees satisfactorily with the other methods of assigning a viscosity

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† 1 P = 10^{-1} Pa s.

to the deep interior. Whether $\dot{\epsilon}$ is fixed by Newtonian or non-Newtonian steady state creep processes is an academic question insofar as the observations of surface movement do not require any close commitment about it. However, I believe it is true to say that the heat transfer process will regulate conditions in which Newtonian processes are collectively going to make a significant contribution to the total creep rate $\dot{\epsilon}$ of the deeply buried planetary material *vis-à-vis* processes like power law creep that imply an infinite effective viscosity as $\dot{\epsilon} \rightarrow 0$. The second point concerns the attempts to use so-called deformation maps. While there have been a number of efforts to put the régime of *in situ* planetary material deformation on such diagrams, the results are very misleading because the boundary value problem is not the one of applying a homogeneous stress or strain rate to planetary material. What happens approximately in the heat-transfer problem is that if any material is too cold to deform the creep resistance is decreased somewhere else to accommodate for it – hence the rather loose talk of ‘rigid ’plates to describe the movements of cold, near-surface material.

Lastly, I would say to those geologists who hope to determine conditions inside the Earth by examining the microstructure of odd bits of rock they find on the surface that they will always have to face the problem of how representative their results are on a much larger scale. At the moment and again largely for historical reasons, such questions are largely brushed aside by assumptions of spherical symmetry, but an understanding of the self regulation of creep resistance inherent in the heat-transfer process for such a large self-gravitating object will soon convince one that intrusive and/or volcanic events only occur *because* a tiny fraction of Earth material can be brought to a very exceptional state of low creep resistance by deformational heating associated with the heat transfer process.

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